





Average Rate of Change

A ratio that describes how one quantity changes as another quantity changes.

We know it as Slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$
Linear functions have a constant rate of change, meaning values increase or decrease at the same rate over a period of time.

Positive R.O.C: increases over time 	Negative R.O.C: decreases over time 
Zero R.O.C: doesn't change over time 	Undefined R.O.C: vertical 

Formula using function notation: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

1. $f(x) = 2x^2 - 3$ from $[2, 4]$.
 $f(2) = 2(2)^2 - 3 = 5$
 $f(4) = 2(4)^2 - 3 = 29$
 $m = \frac{29 - 5}{4 - 2} = \frac{24}{2} = 12$

$(2, 5)$ $(4, 29)$
 x_1, y_1 x_2, y_2

2. $f(x) = -4x + 10$ from $[-1, 3]$.
 $f(-1) = -4(-1) + 10 = 14$
 $f(3) = -4(3) + 10 = -2$
 $m = \frac{-2 - 14}{3 - (-1)} = \frac{-16}{4} = -4$

$(-1, 14)$ $(3, -2)$
 x_1, y_1 x_2, y_2

3. a. Find the rate of change from day 1 to 2.
 $(1, 19)$ $(2, 30)$
 x_1, y_1 x_2, y_2
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 19}{2 - 1} = \frac{11}{1} = 11$

DAYS (X)	AMOUNT OF BACTERIA F(X)
1	19
2	30
3	48
4	76
5	121
6	192

b. Find the rate of change from day 2 to 5.
 $(2, 30)$ $(5, 121)$
 x_1, y_1 x_2, y_2
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{121 - 30}{5 - 2} = \frac{91}{3}$

4. In 2008, about 66 million U.S. households had both landline phones & cell phones. Find the rate of change from 2008 - 2011.

YEAR (X)	HOUSEHOLDS IN MILLIONS F(X)
2008	66
2009	61
2010	56
2011	51

$(2008, 66)$ $(2011, 51)$
 x_1, y_1 x_2, y_2
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{51 - 66}{2011 - 2008} = \frac{-15}{3} = -5$

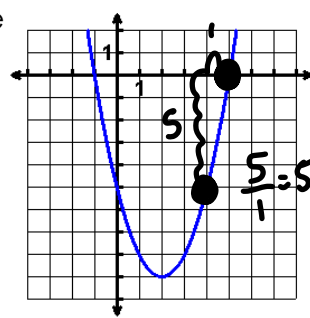
What does this mean?
 That the number of landlines is decreasing by 5 million per year.

5. a. Find the average rate of change from $0 \leq x \leq 1$

$(0, -5)$ $(1, -8)$
 x_1, y_1 x_2, y_2
 $m = \frac{-8 - (-5)}{1 - 0} = \frac{-3}{1} = -3$

b. Find the average rate of change from $4 \leq x \leq 5$

$(4, -5)$ $(5, 0)$
 x_1, y_1 x_2, y_2
 $m = \frac{0 - (-5)}{5 - 4} = \frac{5}{1} = 5$

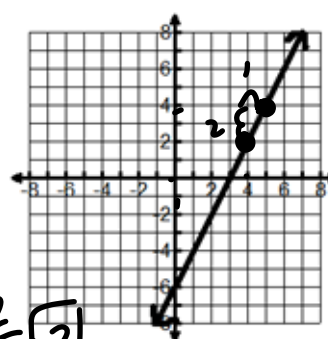


5. Find the average rate of change from $[0, 1]$

$(0, -6)$ $(1, -4)$
 x_1, y_1 x_2, y_2
 $m = \frac{-4 - (-6)}{1 - 0} = \frac{-4 + 6}{1} = 2$

b. Find the average rate of change from $[4, 5]$

$(4, 2)$ $(5, 4)$
 x_1, y_1 x_2, y_2
 $m = \frac{4 - 2}{5 - 4} = \frac{2}{1} = 2$



1. Find the rate of change of Pete's height from 3 to 5 years.

Time (years)	1	2	3	4	5	6
Height(in.)	27	35	37	42	45	49

$(3, 37)$ $(5, 45)$
 x_1, y_1 x_2, y_2
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45 - 37}{5 - 3} = \frac{8}{2} = 4$

2. For $f(x) = -6x - 2$, find the rate of change on the interval $[-2, 4]$.

$f(-2) = -6(-2) - 2 = 12 - 2 = 10$ $(-2, 10)$ $(4, -26)$
 x_1, y_1 x_2, y_2
 $f(4) = -6(4) - 2 = -24 - 2 = -26$
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-26 - 10}{4 - (-2)} = \frac{-36}{6} = -6$

3. For $f(x) = x^2 + 4x + 1$, find the rate of change on the interval $[-2, 4]$.

$f(-2) = (-2)^2 + 4(-2) + 1 = 4 - 8 + 1 = -3$ $(-2, -3)$ $(4, 33)$
 x_1, y_1 x_2, y_2
 $f(4) = (4)^2 + 4(4) + 1 = 16 + 16 + 1 = 33$
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{33 - (-3)}{4 - (-2)} = \frac{36}{6} = 6$

4. You and a friend are trying to decide which theater to go to for a Friday night movie. NCG charges \$7 for the movie ticket and \$3 per food item. Regal's prices are represented by the table.

Write an equation for NCG and Regal. Compare their rates of change and initial cost.

x	g(x)
0	4
1	8
2	12
3	16
4	20

$\left. \begin{matrix} +4 \\ +4 \\ +4 \\ +4 \end{matrix} \right\}$

NCG: $f(x) = 3x + 7$

Regal: $g(x) = 4x + 4$

Characteristic NCG	<, >, or =	Characteristic of Regal
y-intercept of $f(x) = 7$	$>$	y-intercept of $g(x) = 4$
$f(4) = 3(4) + 7 = 19$	$<$	$g(4) = 4(4) + 4 = 20$
Rate of Change of $f(x) = 3$	$<$	Rate of Change of $g(x) = 4$